PID Number: Key Test A

1. (10 points) Use the 4-step process to find $f'(x)$ if

$$f(x) = \frac{-2}{x^2}$$

You can check your answer by using other methods, but you will only receive credit for using the 4-step process.

① $f(x+h) = \dfrac{-2}{(x+h)^2} = \dfrac{-2}{x^2+2xh+h^2}$

② $f(x+h) - f(x) = \dfrac{-2}{x^2+2xh+h^2} - \dfrac{-2}{x^2}$

$$= \frac{-2x^2 + 2(x^2+2xh+h^2)}{(x^2+2xh+h^2)\,x^2}$$

$$= \frac{\cancel{-2x^2} + \cancel{2x^2} + 4xh + 2h^2}{x^2(x^2+2xh+h^2)}$$

③ $\dfrac{1}{h}\left[f(x+h) - f(x)\right] = \dfrac{1}{h} \cdot \dfrac{\cancel{h}(4x+2h)}{x^2(x^2+2xh+h^2)}$

④ $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \dfrac{4x+2\cdot 0}{x^2(x^2+2x\cdot 0+0^2)}$

$$= \frac{4x}{x^4}$$

$$\boxed{f'(x) = \frac{4}{x^3}}$$

2. (15 points) The total amount Michael Jackson has earned from the album *Thriller* is approximated by the function

$$M(x) = \frac{65x^3}{x^3+1}$$

Quotient Rule

Where $M(x)$ is measured in millions of dollars and x is the number of years since the album's release. How fast are the total earnings changing 2 years after the release? (If necessary, round off your answer to the nearest dollar).

$$M'(x) = \frac{65 \cdot 3x^2(x^3+1) - 3x^2(65x^3)}{(x^3+1)^2}$$

$$M'(2) = \frac{65 \cdot 3 \cdot 4(8+1) - 3 \cdot 4 \cdot 65 \cdot 8}{9^2}$$

$$\frac{780}{81}$$

$= 9.6296296$ millions of dollars

$\boxed{= 9{,}629{,}630 \text{ dollars per year}}$

3. (8 points) List the value (or values) of x at which the function $f(x) = \dfrac{x-4}{x^2 - x - 12}$ is not continuous. Use the 3-part definition of continuity to explain why $f(x)$ is not continuous at that point (or those points).

$$f(x) = \frac{x-4}{(x-4)(x+3)}$$

$f(x)$ is not defined at the points $x = 4$ and $x = -3$,

so the function is not continuous at those

points (fails part One).

Note: $\lim_{x \to 4} f(x)$ exists, but the function is still

continuous.

4. (12 points) Find, if they exist, the following limits. If the limits do not exist, show/state why.

(a) $\lim_{x\to(-\infty)} \dfrac{3x^2-5}{7x^5+2x^4-3x} \cdot \dfrac{\frac{1}{x^5}}{\frac{1}{x^5}} = \lim_{x\to(-\infty)} \dfrac{\frac{3}{x^3}-\frac{5}{x^5}}{7+\frac{2}{x}-\frac{3}{x^4}} = \dfrac{0-0}{7+0-0} = \dfrac{0}{7}$

$= \boxed{0}$

(b) $\lim_{x\to(-2)} \dfrac{x^2+3x+2}{x^2-3x-10} = \lim_{x\to(-2)} \dfrac{(x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-5)} = \lim_{x\to(-2)} \dfrac{(x+1)}{(x-5)} = \dfrac{-1}{-7}$

$= \boxed{\dfrac{1}{7}}$

(c) $\lim_{x\to(3)} \dfrac{x^2+3x+2}{x^2-3x-10} = \dfrac{9+9+2}{9-9-10} = \dfrac{20}{-10} = \boxed{-2}$

(d) $\lim_{t\to(3^-)} \dfrac{t^2}{t^2-9}$ approaches $\dfrac{9}{0}$ so it does not exist.

as $t\to 3^-$, t^2 will be slightly less than 9, so the denominator is negative. The numerator is always positive, so $\dfrac{+}{-} = -$ and the limit does not exist since it goes to $-\infty$

5. (10 points) Let

$$f(x) = \begin{cases} 3 - x & \text{if } x < 5 \\ 2 & \text{if } x = 5 \\ 4x - 22 & \text{if } x > 5 \end{cases}$$

Use the three-part definition of continuity at a number to determine whether or not the function f is continuous or discontinuous at $x = 5$. Make sure to explain your answer.

① $f(5) = 2$ ✓

② $\lim_{x \to 5^-} f(x) = 3 - 5 = -2$

$\lim_{x \to 5^+} f(x) = 4 \cdot 5 - 22 = -2$

$\Rightarrow \lim_{x \to 5} f(x) = -2$ ✓

③ $f(5) \stackrel{?}{=} \lim_{x \to 5} f(x)$. No, since $-2 \neq 2$

So the function is not continuous since it fails part 3.

6. (10 points) Find the derivative of

$$y = \frac{3}{\sqrt[5]{x}} + \frac{7}{2x^4} - 18$$

Simplify your answer to the extent of reducing fractions to lowest terms, i.e. $\frac{2}{5x}$ instead of $\frac{4x}{10x^2}$

$$y = \frac{3}{x^{1/5}} + \frac{7}{2}x^{-4} - 18$$

$$y = 3x^{-1/5} + \frac{7}{2}x^{-4} - 18$$

$$y' = 3 \cdot \frac{-1}{5} x^{-6/5} + \frac{7}{\cancel{2}} \cdot (\cancel{-4}^{2}) x^{-5}$$

$$y' = \frac{-3}{5} x^{-6/5} + (-14x^{-5})$$

$$\boxed{y' = \frac{-3}{5} x^{-6/5} - 14x^{-5}}$$

$$= \frac{-3}{5x^{6/5}} - \frac{14}{x^5}$$

7. (10 points) Find the derivative of

$$h(x) = (3x+2)(x^2 - 9x + 6)^8$$

You do not need to simplify your answer.

product rule and chain rule.

$$h'(x) = \frac{d}{dx}(3x+2) \cdot (x^2 - 9x + 6)^8 + (3x+2) \cdot \frac{d}{dx}\left[(x^2 - 9x + 6)^8\right]$$

$$\boxed{h'(x) = 3 \cdot (x^2 - 9x + 6)^8 + (3x+2) \cdot 8(x^2 - 9x + 6)^7 \cdot (2x - 9)}$$

8. (15 points) Find the exact value of the derivative $g'(3)$, if $g(t) = \dfrac{4}{\sqrt{19-5t}}$. Write your answer as a fraction in lowest terms.

$$g(t) = 4 \cdot (19-5t)^{-1/2}$$

$$g'(t) = 4 \cdot \frac{-1}{2} (19-5t)^{-3/2} (-5)$$

$$g'(3) = -2 (19-15)^{-3/2} (-5)$$

$$= 10 \cdot 4^{-3/2}$$

$$= 10 \cdot \frac{1}{8}$$

$$4^{-3/2} = \frac{1}{4^{3/2}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\boxed{g'(3) = \frac{5}{4}}$$

9. (10 points) Find the equation of the tangent line to the graph of

$$f(x) = x^3(x^2 - 1)$$

at the point where $x = 2$. Write your answer in the form $y =$ $68x - 112$

$f'(x) = 3x^2(x^2-1) + 2x \cdot x^3$

$= 3x^4 - 3x^2 + 2x^4$

point-slope formula

$y - y_1 = m(x - x_1)$

4

$f'(x) = 5x^4 - 3x^2$

$m = f'(2) = 5 \cdot 2^4 - 3 \cdot 2^2 = 80 - 12 = 68$

6

$x_1 = 2$

$y_1 = f(2) = 2^3(2^2-1) = 8 \cdot 3 = 24$

$y - 24 = 68(x-2)$

$y = 68x - 136 + 24$

$\boxed{y = 68x - 112}$

Extra Credit: (2 points) Find $f'(x)$ if $f(x) = \pi^2$

$f'(x) = 0$ since π^2 is a constant.